Predicate Logic Exercises 4

Alessandro Rossi

ar279@st-andrews.ac.uk

1 Natural Deduction

Give Natural Deduction proofs to establish the following sequents.

1.1 $\forall x(Px) \vdash_{\mathsf{QL}} \neg\exists x\neg(Px)$

1.2 $\exists x(Px) \vdash_{\mathsf{QL}} \neg\forall x\neg(Px)$

1.3 $\forall x(x = a \rightarrow Fx), Fa \vdash_{\mathsf{QL}} Fa$

1.4 $\vdash_{\mathsf{QL}} \forall x(Px \vee \neg Px)$

1.5 $\vdash_{\mathsf{QL}} \exists x\exists y(x = y)$

1.6 $\vdash_{\mathsf{QL}} \forall x\forall y\forall z((x = y \,\&\, y = z) \rightarrow x = z)$

1.7 $\exists x(Px\&\forall y(Py \rightarrow y = x)) \vdash_{\mathsf{QL}} \forall x\exists y(\neg Px \rightarrow (x \neq y))$

2 More on Natural Deduction

2.1 Existential Quantifier ($\exists$)

- See Read and Wright, *Formal Logic*, p. 208, exercises 1.a - 1.d.

2.1.1 On Existential Introduction ($\exists$I)

- See Tomassi, P. (1999), *Logic*, Ch. 6.III, p. 286.

2.1.2 On Existential Elimination ($\exists$E)

- See Tomassi, P. (1999), *Logic*, Ch. 6.V , pp. 302-303.

2.2 Identity ($=$)

- See Tomassi, P. (1999), *Logic*, Ch. 6.VII, p. 315.

2.3 Solutions

- Solutions to all of the exercises from Read & Wright can be found on pp. 1-2 of my document '10. Natural Deduction' available at: https://ar2797.wixsite.com/rossi/teaching.

2.4 Further Exercises

- See page 2 of my document '12. First-Order Logic' in my webpage.