Propositional Logic Exercises 2

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1 Thinking Semantically

1.1 Logical Properties of Formulae

For each of the following formulae, try to determine its logical status (if it is a tautology, an inconsistency or a contingency) only by thinking about the truth-conditions for the five connectives. Try not to use truth-tables if you can.

1.1.1 $((Q \rightarrow P)\&(P\&\neg P))\&((\neg P \rightarrow Q) \vee (R \vee Q))$;

1.1.2 $(P\&\neg P) \rightarrow (Q \vee (P \leftrightarrow \neg Q))$;

1.1.3 $((P \rightarrow Q)\&Q) \rightarrow P$.

1.2 Sequents

For each of the following sequents, try to determine whether or not it is valid only by thinking about the truth-conditions for the five connectives. Try not to use truth-tables if you can.

1.2.1 $P, \neg P \therefore Q \vee \neg Q$;

1.2.2 $R \vee \neg R \therefore R\&\neg R$;

1.2.3 $P \therefore P$;

1.2.4 $P \rightarrow Q, \neg P \therefore \neg Q$;

1.2.5 $Q \rightarrow \neg Q \therefore \neg Q \rightarrow Q$.

2 Truth-Tables

2.1 Logical Properties of Formulae

Construct truth-tables for the following wffs of PL *and determine whether they are tautologies, inconsistencies or contingencies. Make sure to explain which part of the truth-table establishes your result.*

2.1.1 $(P \rightarrow Q) \vee (Q \rightarrow P)$;

2.1.2 $\neg(P \rightarrow (\neg P \rightarrow Q))$;

2.1.3 $(P \leftrightarrow \neg Q)\&\neg P$;

2.1.4 $\neg\neg P \rightarrow (P \leftrightarrow (Q \vee \neg P))$.

2.2 Sequents

Construct truth-tables for the following sequents of PL *and determine whether or not they are valid. Make sure to explain which part of the truth-table establishes your result.*

2.2.1 $P \rightarrow Q \therefore ((\neg P \vee Q)\&P) \rightarrow Q$;

2.2.2 $\neg P \vee Q \therefore \neg\neg(P\&\neg Q)$;

2.2.3 $Q \rightarrow \neg Q \therefore P \leftrightarrow \neg Q$;

2.2.4 $P \vee Q \rightarrow (P \vee Q) \vee (R\&\neg Q)$.

3 More on Truth-Tables

- For more exercises on testing sequents for validity by truth-tables, see Read & Wright, Formal Logic, p. 34, exercises 1.a - 1.c; 2.a - 2.c; 3.

  For solutions to some of these exercises, see file '5. Language, Models and Truth-Tables' on my webpage: `ar2797.wixsite.com/rossi/teaching`.

- For more exercises on assessing the logical status of a formula, see Read & Wright, Formal Logic, p. 34, exercises 4.a - 4.c.

  For solutions to all of these exercises, see my file '5. Language, Models and Truth-Tables'.

- For more exercises on testing sequents for validity and checking the logical status of a formula, see **Section 3** of my file '5. Language, Models and Truth-Tables'.